TECHNICAL NOTE 398

Compressibility of Natural Gases

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ABSTRACT

The purpose of this paper is to clarify the definition of compressibility and to present a uniform basis upon which instantaneous compressibilities of liquids and gases can be compared. The equations gavering the instantaneous compressibilities of imperfect gases are derived and the concept of pseudo-reduced compressibility is introduced. Part of the data presented by Brown, Katz et al on compressibility factors for natural gases has been rearranged. A graph of pseudo-reduced compressibility vs pseudo-reduced pressure for various pseudo-reduced temperatures is presented. The need for additional work in relating the compressibilities of liquids and gases is discussed.

This information should be of value to reservoir engineers in making non-steady state performance calculations in gas reservoirs. It should be of further use in pointing the direction for additional research in the nature of liquid and gas compressibilities.

INTRODUCTION

With the increasing use of steady and non-steady state well and reservoir data, there is a corresponding increase in the importance of the various factors entering into such calculations. Increasing emphasis is being placed on the necessity for obtaining reasonably accurate estimates of the physical properties of the reservoir fluids well in advance of the more accurate laboratory data.

One such factor is the isothermal coefficient of expansion of the media which are transmitting and attenuating the non-steady state pressure waves. The average isothermal coefficient of expansion, or "compressibility" is a complex function controlled by the physical properties of the formation and the fluids contained therein. The isothermal expansion coefficients for reservoir gases are usually quite variable, in many cases being highly-pressure sensitive. The coefficients for reservoir liquids tend to be pressure sensitive, but not nearly so much as reservoir gases. The coefficients for solids, usually expressed in terms of a "modulus of elasticity" are relatively insensitive to pressure variations within their elastic limits. For this reason, and also because many previous applications have been limited to relatively small pressure ranges, there has been a tendency to ignore the variable nature of isothermal expansion coefficients and treat them as constants.

Also, the term "compressibility" by which these coefficients are generally designated is commonly confused with a similar term, z, used to define the deviation of an imperfect natural gas from the perfect gas laws. A clear distinction should be made at the outset between the term "compressibility", which is an isothermal coefficient of expansion of a substance, and the term "compressibility factor", z, which refers to the deviation of a gas from the perfect gas laws. Although the scope of this paper is limited to the compressibility of single phase natural gases, it is definitely related to the problem of accurately estimating the compressibilities of single phase hydrocarbon reservoir liquids, which will form the basis of a future presentation.

BASIC PRINCIPLES

The coefficient of isothermal compressibility of a substance, c, is usually determined from pressure-volume or pressure-length measurements depending upon whether the substance is single phase gas, liquid, or solid. A convenient method for making such estimates for a finite change in pressure and volume at constant temperature is to use the well known equation

$$c = -\frac{V_1 - V_2}{V_1 (p_2 - p_1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Eq. 1 is negative because the volume of a confined substance decreases as the pressure is increased. In this case $V_1 > V_2$ and $p_2 > p_1$. This equation is useful in approximating the compressibilities of single phase gases and liquids undergoing small pressure changes. It is evident, however, that this equation is almost identical with the determination of Young's modulus of elasticity for solids. If the assumption is made that change in length is directly proportional to change in volume, as would very nearly be the case for a steel rod in tension within its elastic limit, then

$$E_s = -\frac{L_1 (p_2 - p_1)}{L_1 - L_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which E_s is the isothermal expansion coefficient, or Young's modulus of elasticity, for a solid. And further, for this special case

$$c_s = -\frac{L_1 - L_2}{L_1 (p_2 - p_1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

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It is evident, therefore, that Young's modulus for solids is by definition equivalent to the reciprocal of the coefficient of isothermal compressibility in the case of single phase solids within their elastic limit.

Eq. 1, however, is an approximation with definite limitations when the isothermal coefficient of compressibility varies during small pressure changes. It is further limited over large pressure changes by confusion over whether the volume in the denominator should be V_1 or V_2, or some value in between. This objection is easily overcome by expressing Eq. 1 in partial differential form at constant temperature, as follows:

$$c = -\frac{\partial V}{V\,\partial p} \quad \ldots\ldots\ldots\ldots \quad (4)$$

and its reciprocal:

$$E = -\frac{V\,\partial p}{\partial V} \quad \ldots\ldots\ldots\ldots \quad (5)$$

Eqs. 2 and 3 may be similarly expressed, as follows:

$$E_s = -\frac{L\,\partial p}{\partial L} \quad \ldots\ldots\ldots\ldots \quad (6)$$

and

$$c_s = -\frac{\partial L}{L\,\partial p} \quad \ldots\ldots\ldots\ldots \quad (7)$$

Eq. 4 is a general equation which defines the instantaneous coefficient of isothermal compressibility of a substance at a point on an isothermal pressure-volume curve for that substance. If a special case is considered in which c is assumed to be constant, Eq. 4 may be solved, as follows:

$$-\frac{\partial V}{V} = c\,\partial p$$

from which

$$c = \frac{\ln V_1 - \ln V_2}{p_2 - p_1} \quad \ldots\ldots\ldots\ldots \quad (8)$$

Eq. 8, therefore, shows that if the coefficient is actually constant for a given substance, then a graph showing the logarithm of volume, specific volume, density, or formation volume factor vs pressure in psi will be a straight line. Such a graph will indicate a straight line on semi-log graph paper, but not on cartesian coordinate graph paper. Furthermore, if the compressibility is a variable, the pressure-volume curve will not be a straight line on semi-log paper. In this case, however, the value of c at any point on such a curve will be the slope of the curve at that point in log cycles per psi × 2.303. The expansion coefficient, E, will be the cotangent of the curve at that point in psi per log cycle × 0.434.

ISOTHERMAL COMPRESSIBILITY OF GASES

The general expression for the coefficient of isothermal compressibility for gases, c_g, was shown by Muskat[1] from the basic gas equation

$$pV = znRT \quad \ldots\ldots\ldots\ldots \quad (9)$$

when placed in the form of Eq. 4, Eq. 9 at constant temperature becomes

$$c_g = -\frac{\partial V}{V\,\partial p} = \frac{1}{p} - \frac{\partial z}{z\,\partial p}. \quad \ldots\ldots \quad (10)$$

From Eq. 10 it is evident that for perfect gases $z = 1$ and $\partial z/\partial p = 0$, and that in this special case c_g would be inversely proportional to pressure. Natural hydrocarbon gases, however, are not perfect gases. In the low pressure range the compressibility factor, z, decreases as the pressure increases, $\partial z/\partial p$ is negative, and c_g is higher than would be the case for a perfect gas. In the high pressure range, however, z increases, $\partial z/\partial p$ is positive, and c_g is less than would be the case for a perfect gas. Gas compressibilities, therefore, can be determined from PVT laboratory data on actual samples or if such information is lacking, by applying the data of Brown, Katz et al[2] to Eq. 10.

On the other hand, the data in Ref. 2 may be brought into a more direct relation between compressibility, p_r, and T_r by reducing Eq. 10 to dimensionless form. Since $p = p_c \,.\, p_r$, substituting $(p_c \,.\, p_r)$ for p in Eq. 10 reduces same to

$$c_g \,.\, p_c = \frac{1}{p_r} - \frac{\partial z}{z\,\partial p_r} \quad \ldots\ldots\ldots \quad (11)$$

If pressures are measured in psi, the dimensions of c_g are psia.$^{-1}$ The term $c_g \,.\, p_c$, therefore, becomes a dimensionless quantity which is herein defined as the pseudo-reduced compressibility, and

$$c_r = c_g \,.\, p_c \quad \ldots\ldots\ldots\ldots \quad (12)$$

The data in Ref. 2 have been converted to the form of Eq. 11 and are shown graphically on Figs. 1 and 2. Values of c_r and p_r for various fixed values of T_r are shown. It will be noted that the curves for $T_r = 1.05$, and 1.1 are shown as dashed lines. The values of $\partial z/\partial p$ become increasingly difficult to estimate as the critical temperature is approached. For this reason the dashed curves are less reliable than the curves for $T_r > 1.2$. If accuracy greater than that afforded by this chart is desired, laboratory data should be obtained on an actual sample of the gas in question.

EXAMPLE

GIVEN

Reservoir Temperature, °F	195
Reservoir Temperature, °R	655
Reservoir Pressure, psia	4000
Gravity of Reservoir Gas, G*	0.95

FIND

1. Isothermal compressibility coefficient, c_g
2. Isothermal expansion coefficient, E_g

SOLUTION

1. From Ref. 2
   a. $T_c = 436$ °R
   b. $p_c = 644$ psia
   c. $T_r = 1.5$
   d. $p_r = 6.22$
2. From Fig. 2
   a. Using $p_r = 6.22$ and $T_r = 1.5$
   Read Value of $c_r = 0.092$ on graph
   b. Since $c_g p_c = c_r = 0.092$ and $p_c = 644$ psia, then $c_g = 0.092/644$
   $c_g = 143 \times 10^{-6}$ psia^{-1}
   c. $E_g = \frac{1}{c_g} = 6{,}990$ psia

DISCUSSION

If the pressure on the above gas is increased to 8,000 psia, the value of p_r would be 12.44; the corresponding c_r is about 0.0256; and c_g is about 39.8×10^{-6} psia^{-1}. The compressibility coefficients for high pressure gases, therefore, tend to approach values usually associated with reservoir liquids. A comparison of the above values, with approximate values of some other substances, is shown in the following table.

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[1] References given at end of paper.

*Est. from condensate well fluid.

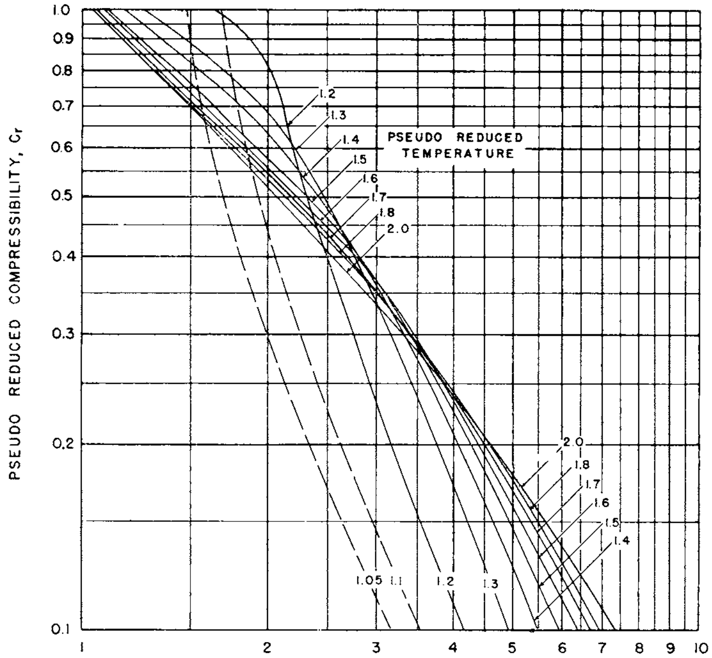

FIG. 1—VARIATION OF REDUCED COMPRESSIBILITY WITH REDUCED PRESSURES FOR VARIOUS FIXED VALUES OF REDUCED TEMPERATURE (FOR c_r BETWEEN 1.0 AND 0.1).

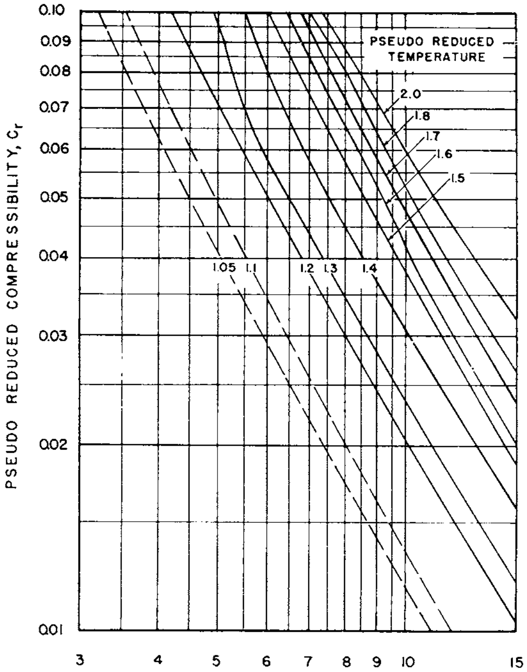

FIG. 2—VARIATION OF REDUCED COMPRESSIBILITY WITH REDUCED PRESSURES FOR VARIOUS FIXED VALUES OF REDUCED TEMPERATURE (FOR c_r BETWEEN 0.1 AND 0.01).

	$c \times 10^{-6}$ psia^{-1}	E psia
Gas at 4,000 psia	143	6,990
Gas at 8,000 psia	39.8	25,100
Reservoir Oil	10	100,000
Water	3	333,000
Sandstone[3]	0.33	3,000,000
Limestone[3]	0.143	7,000,000
Steel[3]	0.033	30,000,000

The expansion coefficient, *E*, indicates that if the hypothetical gas is at a pressure of 4,000 psia, its compressibility corresponds to that of a perfect gas at 6,990 psia. By the same reasoning, the assumed single phase reservoir oil, having a compressibility of 10×10^{-6} psia $^{-1}$, has the same compressibility as a perfect gas at 100,000 psia. The expansion coefficient, *E*, being the reciprocal of the isothermal compressibility, represents the pressure to which a perfect gas must be raised in order to have the same compressibility as the substance under investigation.

CONCLUSIONS

The basic principles underlying the physical determination of instantaneous isothermal coefficients of compressibility and expansion have been outlined. A simplified chart which approximates the isothermal coefficients of compressibility of natural gases has been presented. The fact that the coefficient of expansion, *E*, relates the compressibilities of solids, liquids and gases to that of a perfect gas has been pointed out. With the growing use of compressibility data, there is an increasing need for reasonably accurate data on instantaneous compressibility values on reservoir rocks and liquids, as well as reservoir gases. Average coefficients for complex mixtures of single phase hydrocarbon liquids may be estimated with Standing's data[4]. Instantaneous values for liquids, however, are still determined on an individual basis by actual PVT measurements.

NOMENCLATURE

L = Length, feet
V = Volume, cubic feet
p = Pressure, psia
c = Compressibility, psia^{-1}, $-\dfrac{\partial V}{V\,\partial p}$
E = Expansion coefficient, $1/c$, psia, $-\dfrac{V\,\partial p}{\partial V}$
R = Universal gas constant, per mole
n = Total moles
T = Temperature, °R
z = Gas compressibility factor, $\dfrac{PV}{nRT}$
G = Gas gravity, (Air = 1.0)

SUBSCRIPTS

1 Conditions at a lower pressure
2 Conditions at pressure higher than at condition 1
c Denotes pseudo critical temperature or pressure
r denotes pseudo reduced temperature, pressure, or compressibility
g Denotes single phase gas
L Denotes single phase liquid
s Denotes single phase solid

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