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Compressibility of natural gases

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ABSTRACT

Two current methods for determining the isothermal coefficient of the compressibility of a natural gas are discussed, i.e., the Trube method published in 1957 and the Mattar et al. method published in 1975. The Trube method plots the reduced compressibility $C_r = C_g P_c$ as a function of reduced pressure and temperature, whereas the Mattar et al. procedure plots the function $C_g P_c T_r$, or $C_r T_r$ as a function of reduced pressure and temperature. The Trube chart, which was constructed in 1957 using a graphical method, was recalculated using the Dranchuk and Abou-Kassem eleven factor equation-of-state for the compressibility factor Z. The recalculated Trube chart has a greater range and accuracy than the original chart, particularly in the critical region.

A new procedure for calculating gas compressibility is also presented. This method develops an expression for $C_g P$ dimensionless compressibility as a function of reduced pressure and temperature. The results are presented both graphically and as a subroutine for computer use.

Introduction

It is important to obtain accurate estimates of the physical properties of reservoir fluids in conducting reservoir engineering studies. *PVT* laboratory tests are the main source of physical property data of reservoir fluids. However, when laboratory analyses are not available, other methods for approximating reservoir fluid properties are developed. The isothermal coefficient of compressibility, C_g, of natural gases is one of the important properties that is used in the transient and pseudo-steady state analysis of gas wells.

To develop the expression for the compressibility of natural gas, the real gas law is:

$$PV = ZRT \tag{1}$$

The compressibility factor, Z, is a correction factor to account for the deviation of real gases from ideal gas behavior. Standing and Katz, using the "theorem of corresponding states" correlated the compressibility factor as a function of pseudo-reduced pressure and temperature (Standing and Katz, 1942).

The basic definition for the coefficient of isothermal compressibility C_g, is as follows:

$$C_g = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \tag{2}$$

Muskat combined eqs. 1 and 2 to derive an expression for the isothermal coefficient of compressibility C_g, as a function of the gas compressibility factor Z and pressure (Muskat, 1949):

$$C_g = \frac{1}{P} - \frac{1}{Z}\left(\frac{\partial Z}{\partial P}\right)_T \tag{3}$$

In 1957, Trube introduced the concept of pseudo-reduced compressibility by rewriting Eq. 3 as a function of reduced pressure. Pseudo-reduced compressibility, C_r, is defined as the product of C_g and the pseudo-critical pressure, P_c, of the gas, a dimensionless product (Trube, 1957):

$$C_r = C_g P_c = \frac{1}{P_r} - \frac{1}{Z}\left(\frac{\partial Z}{\partial P_r}\right)_{Tr} \tag{4}$$

Equation 4 provides a direct relationship between the gas compressibility, C_g, and the Standing and Katz compressibility factor chart through the pseudo-reduced pressure and temperature of the gas. Using the Standing and Katz compressibility factor chart and Eq. 4, Trube plotted the reduced compressibility, Cr, as a function of pseudo-reduced pressure and temperature (Trube, 1957). In preparing his chart, Trube used a graphical procedure because in 1957 the Standing and Katz chart had not been reduced to equation form and general purpose computers had not arrived on the scene. As a consequence, Trube's chart is limited to a narrower range of T_r and P_r and its accuracy may be suspect, particularly around the critical pressure and temperature of the gas.

Mattar et al. (1975) presented a somewhat modified chart for natural gas compressibility wherein they plot the product $C_g P_r T_r$ or $C_r T_r$ as a function of P_r and T_r. They employed the same reduced compressibility concept that was derived by Trube (1957). The compressibility factor, Z, and the derivative $(\partial Z/\partial P_r)_{Tr}$ of Eq. 4, however, are obtained from a form of the Benedict, Webb and Ruben equation-of-state (EOS) developed by Dranchuk et al., 1974). This EOS adequately represents the Standing and Katz Z-factor chart, which makes the Mattar et al. chart more consistent and definitely more accurate than Trube's chart especially near the critical pressure and temperature region. The only complication being the introduction of T_r in the compressibility function $C_g P_c T_r$. The Mattar et al. correlation is suitable for computer calculations. A FORTRAN subroutine to perform the compressibility computations and a graphical form of the correlation is given in the original paper (Matter et al., 1975).

Since the advent of the Standing and Katz compressibility factor chart, a number of investigators have expressed it in equation form for computer solution (Grey and Sims, 1959; Sarem, 1961; Papay, 1968; Carlile and Gillet, 1971; Hall and Yarborough, 1973; Brill and Beggs, 1974; Dranchuk et al., 1974; Yarborough and Hall, 1974; Dranchuk and Abou-Kassem, 1975). Takacs reviewed and compared a number of published methods aimed to reproduce the Z-factor chart as a function of pseudo-reduced pressure and temperature (Takacs, 1976). His comparison showed that the Dranchuk and Abou-Kassem eleven factor, generalized Starling EOS method of calculating Z-factors has the smallest absolute error, 0.316% (Dranchuk and Abou-Kassem, 1975). Cox agrees with Takacs (Cox, 1988).

This paper presents a recalculation of Trube's reduced compressibility correlation employing the Dranchuk and Abou-Kassem equation for the Standing and Katz Z-factor chart (Dranchuk and Abou-Kassem, 1975) . A new correlation of dimensionless compressibility of natural gas, $C_g P$, as a function of P_r and T_r, will also be presented. The new correlation employs the Dranchuk and Abou-Kassem Z-factor correlation as well.

Development of the correlation

The general expression for the isothermal coefficient compressibility of real gases, as expressed by Muskat is as follows (Muskat, 1949):

$$C_g = \frac{1}{P} - \frac{1}{Z}\left(\frac{\partial Z}{\partial P}\right)_T \tag{5}$$

By multiplying every term of Eq. 5 by the pressure, P, we get:

$$C_g P = 1 - \frac{P}{Z}\left(\frac{\partial Z}{\partial P}\right)_T \tag{6}$$

Because $P_r = P/P_c$ and $dP = P_c dP_r$, Eq. 6 can be written as follows:

$$C_gP = 1 - \frac{P_r}{Z}\left(\frac{\partial Z}{\partial P_r}\right)_{T_r} \qquad (7)$$

C_gP will be referred to as "dimensionless compressibility".

To obtain correlations for reduced compressibility, C_r, and dimensionless compressibility C_gP, Eqs. 4 and 7 are combined with the Dranchuk and Abou-Kassem expression of the Z-factor chart:

$$Z = 1 + \left(A_1 + \frac{A_2}{T_r} + \frac{A_3}{T_r^3} + \frac{A_4}{T_r^4} + \frac{A_5}{T_r^5}\right)\rho_r + \left(A_6 + \frac{A_7}{T_r} + \frac{A_8}{T_r^2}\right)\rho_r^2 - A_9\left(\frac{A_7}{T_r} + \frac{A_8}{T_r^2}\right)\rho_r^5 + A_{10}\left(1 + A_{11}\rho_r^2\right)\frac{\rho_r^2}{T_r^3}\exp(-A_{11}\rho_r^2) \qquad (8)$$

where: $A_1 = 0.3265$, $A_2 = -1.0700$, $A_3 = -0.5339$, $A_4 = 0.01569$, $A_5 = -0.05165$, $A_6 = 0.5475$, $A_7 = -0.7361$, $A_8 = 0.1844$, $A_9 = 0.1056$, $A_{10} = 0.6134$, $A_{11} = 0.7210$.

The Z term of the Eqs. 4 and 7 can be directly determined from Eq. 8. To determine the partial derivative $(\partial Z/\partial P_r)_{Tr}$ from the derivative of Eq. 8, however, we need to relate it to the partial derivative $(\partial Z/\partial \rho_r)_{Tr}$. These two partial derivatives can be related using the reduced density equation ρ_r (Mattar et al., 1975):

$$\rho_r = \frac{0.27P_r}{ZT_r} \qquad (9)$$

The critical value of Z was assumed to be 0.27 (Dranchuk and Abou-Kassem, 1975). Now, by differentiating Eq. 9:

$$\left(\frac{\partial Z}{\partial P_r}\right)_{T_r} = \frac{0.27\,(\partial Z/\partial \rho_r)_{T_r}}{ZT_r + \rho_r T_r(\partial Z/\partial \rho_r)_{T_r}} \qquad (10)$$

and substituting $(\partial Z/\partial P_r)_{Tr}$ in Eqs. 4 and 7, we get the reduced compressibility C_r and the dimensionless compressibility, C_gP, equations in terms of Z and $(\partial Z/\partial \rho_r)_{T_r}$:

$$C_r = C_gP_c = \frac{1}{P_r} - \frac{1}{ZT_r}\left(\frac{0.27(\partial Z/\partial \rho_r)_{T_r}}{Z + \rho_r(\partial Z/\partial \rho_r)_{T_r}}\right) \qquad (11)$$

$$C_gP = 1 - \frac{P_r}{ZT_r}\left(\frac{0.27(\partial Z/\partial \rho_r)_{T_r}}{Z + \rho_r(\partial Z/\partial \rho_r)_{T_r}}\right) \qquad (12)$$

$(\partial Z/\partial \rho r)_{T_r}$ can be evaluated from Eq. 8 after differentiation:

$$\left(\frac{\partial Z}{\partial \rho_r}\right)_{T_r} = \left(A_1 + \frac{A_2}{T_r} + \frac{A_3}{T_r^3} + \frac{A_4}{T_r^4} + \frac{A_5}{T_r^5}\right) + 2\left(A_6 + \frac{A_7}{T_r} + \frac{A_8}{T_r^2}\right)\rho_r - 5A_9\left(\frac{A_7}{T_r} + \frac{A_8}{T_r^2}\right)\rho_r^4 + \left(\frac{2A_{10}\rho_r}{T_r^3} + 2A_{10}A_{11}\frac{\rho_r^3}{T_r^3} - 2A_{10}A_{11}^2\frac{\rho_r^5}{T_r^3}\right) \times \exp(-A_{11}\rho_r^2) \qquad (13)$$

Finally, the natural gas compressibility can be determined from the reduced compressibility, or from the dimensionless compressibility as follows:

$C_g = C_r/P_c$, or $C_g = C_gP/P$

Presentation of the correlation

Equations 8, 9, 11 and 13, or Eqs. 8, 9, 12 and 13 are the essential mathematical expressions for the determination of natural gases compressibility factor, Z, and the isothermal coefficient of compressibility, C_g.

(1) **Recalculation of Trube's compressibility correlation.** The reduced compressibility, C_r, correlation developed by Trube has been recalculated using the Dranchuk and Abou-Kassem form of the Starling eleven factor equation-of-state. The correlation is presented in Fig. 1.

The correlation was checked against Trube's original chart and the Mattar et al. correlations. It was found to be very close to the Mattar et al. correlation. The minor differences

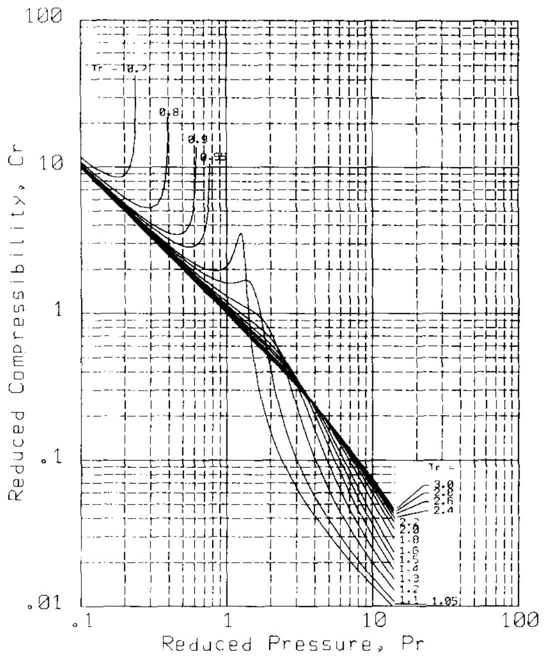

Fig. 1. Reduced compressibility as a function of reduced pressure and temperature.

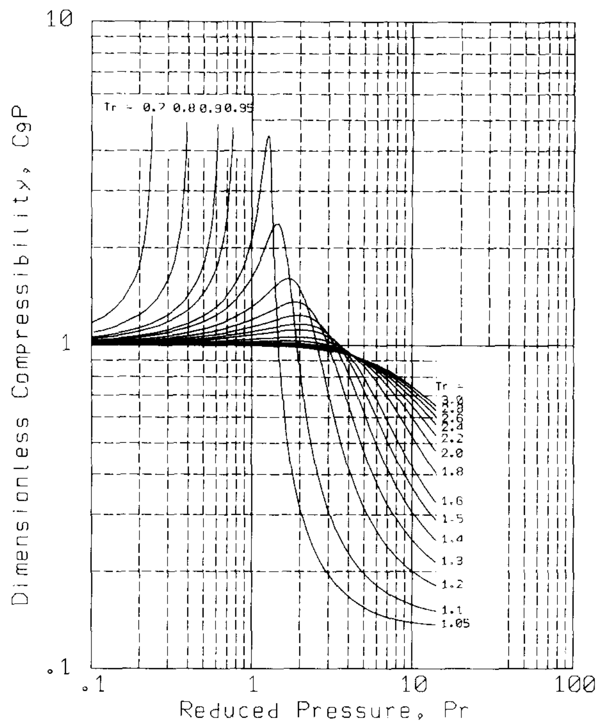

Fig. 2. Dimensionless compressibility as a function of reduced pressure and temperature.

noticed in the results were thought to be due to the more accurate EOS model of the Z-factor chart (Dranchuk and Abou-Kassem, 1975) used.

The recalculated Trube's correlation presented in this paper has a wider range of applicability than the old correlation of Trube's and also should be more reliable particularly in the critical region.

(2) **The dimensionless compressibility, C_gP, correlation.**Equation 6, with the Dranchuk and Abou-Kassem EOS was used to develop a correlation for natural gas compressibility. The chart is presented in Fig. 2. The correlation is found to be easy to use and perhaps a little less ambiguous when compared to the reduced compressibility correlation and the Mattar et al. correlation.

The expression for the determination of natural gas compressibility factor, Z, using the Dranchuk and Abou-Kassem model and gas compressibility, C_g, through the dimensionless compressibility term can be implemented in a spreadsheet or a computer program subroutine. A listing of the BASIC subroutine is given in Appendix A. The pseudo critical temperature and pressure of the natural gas and the pressure and temperature in question need to be inputted to the subroutine. These data, however, can be inputted through a main program, if this subroutine is implemented with others to perform a more complicated operation than just the determination of compressibility properties of the natural gas.

Conclusions

(1) Trube's reduced compressibility, C_r, chart is recalculated, using a more accurate model of Standing and Katz's Z-factor chart. The new chart also has a wider range of applicability.

(2) A new correlation, dimensionless compressibility, is presented. This correlation is the simplest of the three correlations relating compressibility to the pseudo-reduced properties of

the gas, and is less ambiguous than the two previously published correlations of compressibility. The recalculated Trube's correlation, and the dimensionless compressibility correlation all give equivalent results.

(3) The dimensionless compressibility correlation is presented both graphically and as a subroutine for computer use.

Epilogue and acknowledgement

The original concept of dimensionless compressibility, C_gP, was suggested by Dr. H.J. Ramey, Jr. After this subject paper was written the authors were made aware, in a discussion with Dr. Ramey, that there existed a Master of Science report at Stanford University entitled "The Isothermal Compressibility of Natural Gases" by Ronald Pantin, January 1977 (Pantin, 1977). In his report Pantin developed the equation for dimensionless compressibility. Using the same form of the Benedict Webb and Ruben equation-of-state as used by Mattar et al., Pantin developed a graph of dimensionless compressibility as a function of P_r and T_r. In addition,he also presents a listing of a subroutine written in FORTRAN for determining the compressibility factor Z, and the dimensionless compressibility, C_gP.

The Pantin Master of Science report was supervised by Dr. H.J. Ramey, Jr. and Dr. M.B. Standing. Dr. Ramey encouraged us to submit this paper for publication.

The authors wish to acknowledge and recognize Drs. Ramey and Standing and Ronald Pantin as the originators of the concept of dimensionless compressibility, C_gP in terms of pseudo-reduced pressure and temperature.

Nomenclature

A_1–A_{11}	Coefficients of the generalized equation-of-state
C_g	Gas isothermal compressibility (Psia^{-1})
C_r	Reduced compressibility (dimensionless)
P	Pressure (Psia)
Pc	Pseudo-critical pressure
P_r	Pseudo-reduced pressure (dimensionless)
R	Universal gas constant
T	Temperature (°R)
T_r	Pseudo-reduced temperature (dimensionless)
V	Volume (ft^3)
Z	Gas compressibility factor (dimensionless)
ρ_r	Reduced density (dimensionless)

SI metric conversion factors

Psi$\times$6.894757 E+00=$\boldsymbol{KP_a}$
Cuft$\times$2.831685 E−02=m^3
R=1.8 K

Appendix A

```
10  REM .. NGCOMP SUBROUTINE
20  REM .. -----------------
30  REM ..
40  REM .. NGCOMP is a subroutine designed to calculate
50  REM .. natural gases compressibility factor, Z-Factor,
60  REM .. using Dranchuk and Abou-Kassem EOS correlation
70  REM .. of the Standing and Katz Z-Factor chart. It also
80  REM .. calculates gases isothermal coefficient of
90  REM .. expansion, Cg, from the dimensionless
100 REM .. compressibility correlation at any desired
110 REM .. pressure and temperature. The input to this
120 REM .. subroutine is the pseudo critical properties
130 REM .. of the natural gas and the pressure and
140 REM .. temperature at which the above properties need
150 REM .. to be determined.
160 REM
170 REM
180 REM .. DEFINITION OF VARIABLES
190 REM .. -----------------------
200 REM ..
210 REM .. P  - Pressure, Psia or Kpa
220 REM .. T  - Temperature, deg. Rankine or Kelvin
230 REM .. PC - Pseudo Critical Pressure, Psia or Kpa
240 REM .. TC - Pseudo Critical Temp., deg. Rankine or Kelvin
250 REM .. PR - Pseudo reduced Pressure, dimensionless
260 REM .. TR - Pseudo reduced Temperature, dimensionless
270 REM .. DR - Pseudo reduced Density, dimensionless
280 REM .. Z  - Gas Compressibility Factor, dimensionless
290 REM .. IFLAG - Iteration Flag
300 REM .. IF IFLAG = 0 Either or Both PR and TR are
310 REM .. Outside the Range of the Dranchuk and
320 REM .. Abou-Kassem EOS correlation.
330 REM .. IF IFLAG > 0 Then it is the Number of
340 REM .. Iterations it took to correlate Z Factor.
350 REM ..
360 REM ..
370 DIM A(11)
380 DATA 0.3265,-1.07,-0.5339,0.01569,-0.05165
390 DATA 0.5475,-0.7361,0.1844,0.1056,0.6134,0.721
400 REM ..
410 REM ..
420 REM .. Reading the eleven factors of the EOS
430 FOR I =1 TO 11
440 READ A(I)
450 NEXT I
460 REM .. Inputing the required data
470 PRINT"Input S if you want to use SI units"
480 PRINT"Input P if you want to use Practical units"
490 INPUT CHOICE$
500 IF CHOICE$="S" OR CHOICE$="s" THEN 530
510 IF CHOICE$="P" OR CHOICE$="p" THEN 650
520 GOTO 470
530 PRINT "Input Gas Critical Pressure in Kpa"
540 INPUT PC
550 PRINT
560 PRINT "Input Gas Critical Temperature in deg. Kelvin"
570 INPUT TC
580 PRINT
590 PRINT "Input Gas Pressure in Kpa"
600 INPUT P
610 PRINT
620 PRINT "Input Gas Temperature in deg. Kelvin"
630 INPUT T
640 GOTO 760
650 PRINT "Input Gas Critical Pressure in Psia"
660 INPUT PC
670 PRINT
680 PRINT "Input Gas Critical Temperature in deg. Rankine"
```

```
690 INPUT TC
700 PRINT
710 PRINT "Input Gas Pressure in Psia"
720 INPUT P
730 PRINT
740 PRINT "Input Gas Temperature in deg. Rankine"
750 INPUT T
760 CLS
770 REM ..
780 REM .. Calculation of Certain Variables
790 REM ..
800 IFLAG = 0
810 J = 1
820 DR = 1
830 PR = P / PC
840 TR = T / TC
850 TR2 = TR ^ 2
860 TR3 = TR ^ 3
870 TR4 = TR ^ 4
880 A = A(1) * TR + A(2) + A(3) / TR2
890 B = A(4) / TR3 + A(5) / TR4
900 C0 = A + B
910 C1 = A(7) + A(8) / TR
920 C2 = A(6) * TR + C1
930 C3 = - C1 * A(9)
940 C4 = A(10) / TR2
950 REM ..
960 REM ..
970 REM .. Check if Data are Out of Range
980 REM ..
990 IF PR > 30 THEN 1780
1000 IF TR >= 1 THEN 1060
1010 REM ..
1020 J = 0
1030 DR = 0
1040 DELDR = .1
1050 REM ..
1060 IF TR >= 3 THEN 1780
1070 IF TR = 1 THEN 1780
1080 REM ..
1090 REM ..
1100 REM .. Solution of EOS for Z-Factor
1110 REM ..
1120 FOR IFLAG = 1 TO 100
1130 IF J > 0 THEN 1160
1140 DR1 = DR
1150 DR = DR + DELDR
1160 DR2 = DR ^ 2
1170 DR4 = DR ^ 4
1180 DR5 = DR ^ 5
1190 REM ..
1200 T1 = C0 * DR
1210 T2 = C2 * DR2
1220 T3 = C3 * DR5
1230 T4 = C4 * DR2
1240 T5 = A(11) * DR2
1250 T6 = EXP(-T5)
1260 REM ..
1270 AA = ( TR + T1 + T2 + T3 ) * DR
1280 BB = T4 * DR * ( 1 + T5 ) * T6
1290 ZCPR = AA + BB
1300 AAA = TR + 2 * T1 + 3 * T2 + 6 * T3
1310 BBB = T4 * T6 * ( 3 + 3 * T5 - 2 * T5 ^ 2 )
1320 DP = AAA + BBB
1330 REM ..
1340 REM ..
1350 IF J > 0 THEN 1430
1360 PRCAL = ZCPR / .27
1370 IF (ABS ( PRCAL - PR )) <= .001 THEN 1510
1380 IF PRCAL < PR THEN 1500
1390 DR = DR1
1400 DELDR = DELDR / 2
1410 GOTO 1500
1420 REM ..
1430 DR1 = DR - ( ZCPR - .27 * PR ) / DP
1440 IF DR1 > 0 THEN 1460
1450 DR1 = .5 * DR
1460 IF DR1 <= 2.2 THEN 1480
1470 DR1 = DR + .9 * ( 2.2 - DR )
1480 IF ( ABS ( DR - DR1 ) ) <= .00001 THEN 1510
1490 DR = DR1
1500 NEXT IFLAG
1510 Z = .27 * PR / ( DR * TR )
1520 REM ..
1530 REM .. Calculation of the Dimensionless Compressibility
1540 REM ..  and the Gas Compressibility, Cg
1550 REM ..
1560 X = ( T1 + 2 * T2 + 5 * T3 ) / ( TR * DR )
1570 Y = (2 * T4 + 2 * T4 * T5 - 2 * T4 * T5 ^ 2)
1580 YY = Y * T6 / ( TR * DR )
1590 DZDDR = X + YY
1600 REM ..
1610 E = ( PR / Z ) * ( .27 * DZDDR )
1620 EE = ( Z * TR + DR * TR * DZDDR )
1630 CGP = 1 - E / EE
1640 IF CGP < 0 THEN 1780
1650 REM ..
1660 CG = CGP / P
1670 PRINT "Under Pressure = ";P;
1680 IF CHOICE$="S" OR CHOICE$="s" THEN PRINT "Kpa";
     ELSE PRINT "Psia";
1690 PRINT" and Temperature = ";T;
1700 IF CHOICE$="S" OR CHOICE$="s" THEN PRINT "deg. Kelvin"
     ELSE PRINT "Deg. Rankine"
1710 PRINT
1720 PRINT "The Natural Gas Has";
1730 PRINT " a Compressibility Factor, Z = ";Z
1740 PRINT
1750 PRINT "and a Compressibility coefficient, Cg = ";CG;
1760 IF CHOICE$="S" OR CHOICE$="s" THEN PRINT "1/Kpa";
     ELSE PRINT "1/Psia";
1770 GOTO 1800
1780 PRINT "Data are out of range"
1790 RETURN
1800 END
```

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