When an adiabatic irreversible expansion or compression becomes reversible

2009 Eur. J. Phys. 30 487

(http://iopscience.iop.org/0143-0807/30/3/007)

IOP PUBLISHING EUROPEAN JOURNAL OF PHYSICS

Eur. J. Phys. **30** (2009) 487–495 doi:10.1088/0143-0807/30/3/007

When an adiabatic irreversible expansion or compression becomes reversible

Joaquim Anacleto^{1,2,3}, **J M Ferreira**^{1} **and A A Soares**^{1}

[1] Departamento de Física, Universidade de Trás-os-Montes e Alto Douro, Apartado 1013, P-5001-801 Vila Real, Portugal
[2] IFIMUP and IN—Institute of Nanoscience and Nanotechnology, Departamento de Física da Faculdade de Ciências da Universidade do Porto, Rua do Campo Alegre, 687, P-4169-007 Porto, Portugal

E-mail: anacleto@utad.pt, jferreir@utad.pt and asoares@utad.pt

Received 8 January 2009, in final form 16 February 2009
Published 20 March 2009
Online at stacks.iop.org/EJP/30/487

Abstract

This paper aims to contribute to a better understanding of the concepts of a *reversible process* and *entropy*. For this purpose, an adiabatic irreversible expansion or compression is analysed, by considering that an ideal gas is expanded (compressed), from an initial pressure P_{i} to a final pressure P_{f}, by being placed in contact with a set of N work reservoirs with pressures decreasing (increasing) in a geometric or arithmetic progression. The gas entropy change ΔS is evaluated and it is clearly shown that $\Delta S > 0$ for any finite N, but as the number of work reservoirs goes to infinity the entropy change goes to zero, i.e. the process becomes reversible. Additionally, this work draws attention to the work reservoir concept, which is virtually ignored in the literature, and to its analogy with the commonly used heat reservoir concept. Finally, it complements and reinforces an earlier study dealing with irreversible cooling or heating so that the synergy created by the two studies is important from both theoretical and educational standpoints.

1. Introduction

Thermodynamics is an area of physics where subtleties abound. Although the mathematics required to deal with its laws and concepts is not complex, the aforementioned subtleties make thermodynamics a difficult teaching and learning subject. These difficulties have, however, stimulated interesting discussions [1–11] whose aim is to make thermodynamics unambiguous and attractive, thus reinforcing its formative role for physics and engineering students.

---

[3] Author to whom any correspondence should be addressed.

The concept of entropy seems to be the most difficult one to grasp [10, 11], possibly because its physical meaning is closely related to other difficult concepts, such as that of reversibility.

Miranda [10] undertook an interesting study dealing with the transition from an irreversible process characterized by abrupt cooling or heating to a reversible process characterized by gradual cooling or heating, between an initial temperature T_i and a final temperature T_f, i.e. he showed that irreversibility can be successively reduced as the temperature differences between the system and the heat reservoirs become arbitrarily small. In other words, since the heat exchange involving finite differences in temperature always causes irreversibility, to achieve reversibility one must reduce such differences to infinitesimal values, by using an infinite number of heat reservoirs.

However, adiabatic processes can also experience irreversibility. For these processes there is, by definition, no heat exchange and irreversibility is due to abrupt work exchanges, i.e. abrupt compressions or expansions. Similar to the heat exchange case where the concept of heat reservoir is used to describe the process, it is useful to consider the work exchange as an interaction with one or several work reservoirs. The work reservoir concept, which is almost neglected in the literature, is defined here as a system that exchanges work at a constant pressure, so that all processes *within* it are reversible [2, 12].

As all the processes considered in this work are adiabatic ($\delta Q = 0$) and irreversible, no entropy change in surroundings occurs and the system entropy increases, as is required by the second law.

Even though adiabatic processes have been extensively discussed in the literature [1, 7, 13–15] and are featured in thermodynamics textbooks, e.g. [12, 16, 17], the situations analysed in the present work have, as far as we know, been ignored. Thus, the present work aims to fill this gap in the existing literature by providing a new insight into the concepts of entropy and reversibility. It also aims to provide an essential complement to a previous work [10] in the sense that, in conjunction with the latter, it enables an understanding of the two main causes of irreversibility: heat exchange through finite temperature differences and work exchange through finite pressure differences.

This paper is intended for advanced students, who will derive additional benefit by studying this work in conjunction with a previous one [10]. It is believed that this synergetic approach is of a significant didactical value and contributes to a deeper understanding of the concepts of reversible processes and entropy, both of which are of major relevance to the area of physics education.

2. Abrupt adiabatic compressions or expansions

Let us consider a system that can be characterized by three variables, namely the pressure P exerted by the system on its surroundings, the volume V and the temperature T. These systems are commonly called PVT systems [16].

As several publications have shown [1, 2, 13], a thermodynamical interaction between a PVT system and its surroundings can be described by the following relation:

$$T\,\mathrm{d}S - P\,\mathrm{d}V = -T_\mathrm{e}\,\mathrm{d}S_\mathrm{e} + P_\mathrm{e}\,\mathrm{d}V_\mathrm{e}, \tag{1}$$

where S is the entropy and the subscript e denotes a surroundings (external) variable.

From (1), we can get the definitions of heat (Q), work (W), configuration work (W_C) and dissipative work (W_D), as in [13]:

$$\delta Q = -T_\mathrm{e}\,\mathrm{d}S_\mathrm{e} \tag{2}$$

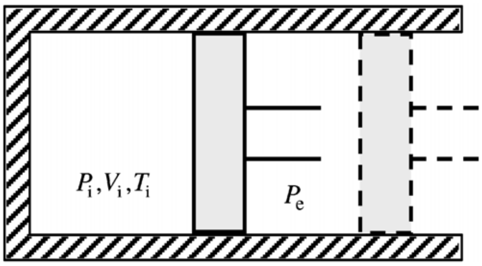

Figure 1. An adiabatic cylinder with an adiabatic piston contains a monatomic ideal gas. The piston allows the gas to interact with its external environment whose pressure P_e is kept constant. The initial state is (P_i, V_i, T_i) and when the piston is released, the gas reaches a final state whose pressure equals P_e, i.e. $P_f = P_e$.

$$\delta W = P_e \, dV_e \tag{3}$$

$$\delta W_C = -P \, dV \tag{4}$$

$$\delta W_D = P_e \, dV_e + P \, dV. \tag{5}$$

For an adiabatic process we have by definition that $\delta Q = 0$ and thus, from equations (1) and (2), the differential of the system entropy is given by

$$dS = \frac{P \, dV}{T} + \frac{P_e \, dV_e}{T}. \tag{6}$$

In this study, we will consider an experimental situation where the system is an ideal gas enclosed in an adiabatic cylinder with an adiabatic and frictionless piston, as illustrated in figure 1. The gas can interact with its external environment only by exchanging work.

Initially, the piston is fixed and the gas is in an equilibrium state (P_i, V_i, T_i). The external pressure P_e is different from the initial gas pressure, i.e. $P_e \neq P_i$, and is kept constant. When the piston is released, an abrupt process of expansion or compression takes place till the gas pressure within becomes equal to the external pressure and the gas reaches the final state $(P_f = P_e, V_f, T_f)$.

For an ideal gas, the following relations hold:

$$PV = nRT, \tag{7}$$

$$dU = nc_V \, dT, \tag{8}$$

where n is the amount of gas, $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$ is the universal gas constant, U is the internal energy and c_V is the constant volume molar heat capacity.

Since the process is adiabatic, using the first law and (3), equation (8) can be written as

$$P_e \, dV_e = nc_V \, dT. \tag{9}$$

Using (7) and (9), an appropriate manipulation of equation (6) gives

$$\frac{dS}{nR} = \frac{1}{a}\frac{dT}{T} - \frac{dP}{P}, \tag{10}$$

where $a = R/c_P$ and $c_P = c_V + R$ is the constant pressure molar heat capacity. By integrating equation (10), we obtain

$$\frac{\Delta S}{nR} = \frac{1}{a} \ln\left(\frac{T_f}{T_i}\right) - \ln\left(\frac{P_f}{P_i}\right). \tag{11}$$

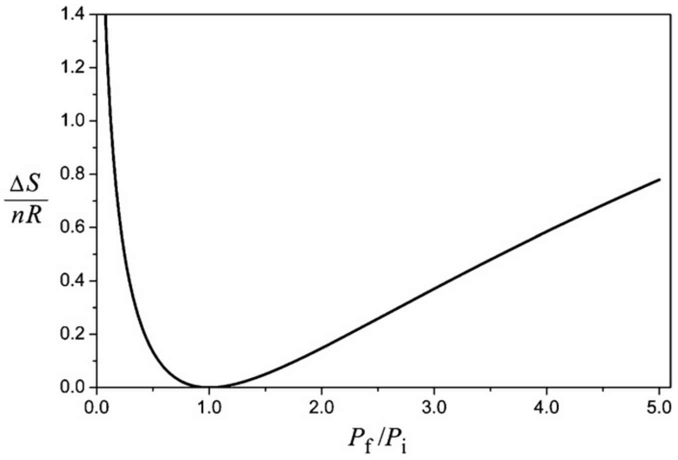

Figure 2. Graph of equation (13) for a monatomic ideal gas ($a = 0.4$). We observe that $\Delta S \geqslant 0$, where the equality holds, as expected, when $P_f = P_i$, i.e. for reversible expansions or compressions. In addition, $\Delta S \to +\infty$ as $P_f/P_i \to 0$ or as $P_f/P_i \to +\infty$.

Taking into account that $dV_e = -dV$ and $P_f = P_e$, it is easy to show that the quotient T_f/T_i in the above equation can be written as

$$\frac{T_f}{T_i} = 1 + a\left(\frac{P_f}{P_i} - 1\right), \tag{12}$$

which combined with (11) finally leads to

$$\frac{\Delta S}{nR} = \frac{1}{a}\ln\left[1 + a\left(\frac{P_f}{P_i} - 1\right)\right] - \ln\left(\frac{P_f}{P_i}\right). \tag{13}$$

Equation (13) gives us the normalized entropy variation for an adiabatic abrupt change in pressure, for an ideal gas. Interestingly enough ΔS is always non-negative, regardless of whether the process is a compression or an expansion, which can be seen by plotting $\Delta S/(nR)$ as a function of P_f/P_i for a monatomic gas ($a = 0.4$), as shown in figure 2. This fact has a consequence worth mentioning: it is not possible to build a cyclic process with only abrupt adiabatic expansions and compressions. When the pressure adiabatically and abruptly changes from P_i to P_f, and then back from P_f to P_i, in both processes the system entropy increases.

This fact contrasts with the abrupt cooling or heating processes described in [10]. Indeed, for these processes, considering for comparison purposes the system as a monatomic ideal gas kept at constant volume, the normalized entropy variation is given by [10, 16]

$$\frac{\Delta S}{nR} = \frac{3}{2}\ln\left(\frac{T_f}{T_i}\right), \tag{14}$$

from which we see that ΔS decreases for cooling and increases for heating, thus allowing us to build cycles with such processes. Figure 3 compares the graphs of equations (13) and (14).

In the following sections, we analyse situations in which the gas pressure varies from a given P_i to a given P_f, by placing it into contact with a succession of work reservoirs with decreasing (expansion) or increasing (compression) pressures. A conceptually similar situation—but involving heat reservoirs instead—is usually studied qualitatively in the classroom and in thermodynamics textbooks, and a quantitative sketch can be found in [12], but a more comprehensive study of the problem is undertaken in [10].

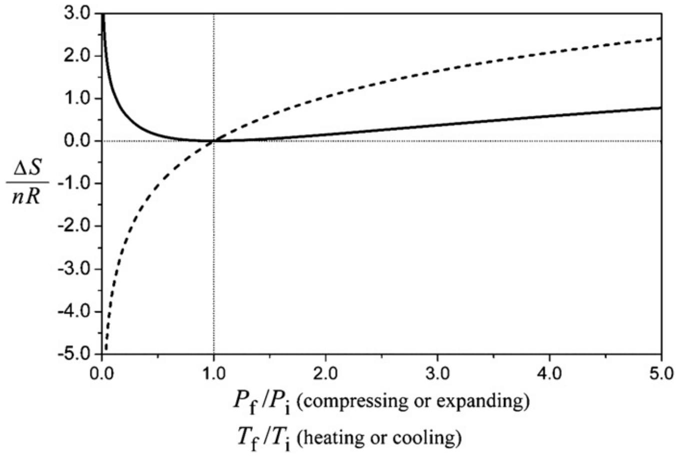

Figure 3. Variation of normalized entropy for a monatomic gas as a function of (i) P_f/P_i (full line) and (ii) T_f/T_i (dashed line). ΔS is always positive for abrupt adiabatic expansions ($P_f < P_i$) or compressions ($P_f > P_i$), in contrast with abrupt cooling ($T_f < T_i$) and heating ($T_f > T_i$), where ΔS changes from negative to positive, respectively. As expected, $\Delta S = 0$ when $P_f = P_i$ or when $T_f = T_i$. In addition, $\Delta S \to +\infty$ as $P_f/P_i \to 0$, as $P_f/P_i \to +\infty$ and as $T_f/T_i \to +\infty$, and $\Delta S \to -\infty$ as $T_f/T_i \to 0$.

However, as already mentioned, the case of work reservoirs is absent from the literature and two different situations are considered in this context: (i) the pressures of the work reservoirs are in a geometric progression and (ii) they are in an arithmetic progression. In both these situations, the system entropy changes can be evaluated in an explicit way. It will be shown that such entropy changes (which are also the total entropy changes, since the processes are adiabatic) are always positive provided the number of work reservoirs is finite. However, as this number increases indefinitely the entropy changes tend to zero, i.e. the process becomes reversible.

3. Expanding (or compressing) in a geometric progression

The initial gas state is characterized by (P_i, T_i, V_i). If the external pressure is $P_e \neq P_i$, i.e. if the system is in contact with a work reservoir P_e, the final gas pressure will be

$$P_f = P_e. \tag{15}$$

As the gas pressure changes abruptly, the process is irreversible and the entropy variation is given by (13). The condition for having an expansion is $P_f < P_i$ and for having a compression is $P_f > P_i$.

Let us now imagine a set of N work reservoirs, whose pressures, $P_1, P_2, \ldots, P_{N-1}$, $P_N = P_f$, are in a geometric progression with a ratio α, i.e.

$$P_1 = \alpha P_i, \qquad P_2 = \alpha^2 P_i, \ldots, \; P_f = \alpha^N P_i. \tag{16}$$

It is obvious that for the case of an expansion $\alpha < 1$ and for the case of a compression $\alpha > 1$. When the system at pressure P_{k-1} is put into contact with the reservoir at pressure P_k, the entropy change in the system is given by (13), i.e.

$$\frac{\Delta S_k}{nR} = \frac{1}{a}\ln\left[1 + a\left(\frac{P_k}{P_{k-1}} - 1\right)\right] - \ln\left(\frac{P_k}{P_{k-1}}\right). \tag{17}$$

Thus, the total change in entropy, when the system goes from pressure P_i to P_f, is

$$\frac{\Delta S}{nR} = \frac{1}{a}\sum_{k=1}^{N}\ln\left[1 + a\left(\frac{P_k}{P_{k-1}} - 1\right)\right] - \sum_{k=1}^{N}\ln\left(\frac{P_k}{P_{k-1}}\right). \tag{18}$$

In the above sums, $P_0 = P_\mathrm{i}$ and $P_N = P_\mathrm{f}$. From (16), $P_k/P_{k-1} = \alpha$ and $\alpha^N = P_\mathrm{f}/P_\mathrm{i}$. Therefore, we can write (18) as

$$\frac{\Delta S}{nR} = \frac{1}{a}\sum_{k=1}^{N}\ln[1 + a(\alpha - 1)] - \sum_{k=1}^{N}\ln(\alpha). \tag{19}$$

The evaluation of the above sums is straightforward and leads to

$$\frac{\Delta S}{nR} = \frac{1}{a}N\ln\left[1 + a\left(\left(\frac{P_\mathrm{f}}{P_\mathrm{i}}\right)^{1/N} - 1\right)\right] - \ln\left(\frac{P_\mathrm{f}}{P_\mathrm{i}}\right). \tag{20}$$

This equation gives us the entropy change as a function of N (the number of work reservoirs). The next step is to evaluate this change in the case of an infinite number of work reservoirs. Introducing the auxiliary variables $\beta = (P_\mathrm{f}/P_\mathrm{i})$ and $x = 1/N$, from (20), we can write

$$\lim_{N\to\infty}\left(\frac{\Delta S}{nR}\right) = \frac{1}{a}\lim_{x\to 0^+}\frac{\ln[1 + a(\beta^x - 1)]}{x} - \ln(\beta). \tag{21}$$

Applying L'Hôpital's rule, we obtain

$$\lim_{N\to\infty}\left(\frac{\Delta S}{nR}\right) = \frac{1}{a}\lim_{x\to 0^+}\frac{a\beta^x\ln\beta}{1 + a(\beta^x - 1)} - \ln(\beta). \tag{22}$$

The above limit is now trivial and leads to

$$\lim_{N\to\infty}\left(\frac{\Delta S}{nR}\right) = \ln(\beta) - \ln(\beta) = 0. \tag{23}$$

The above result clearly shows that for the case of an infinitely large number of work reservoirs, the system entropy change is exactly zero, i.e. the process has become reversible. Therefore, the meaning of a reversible expansion or compression is as follows: the number of work reservoirs with which the system interacts is arbitrarily large.

In order to get some feeling for this situation, let us consider a numerical example. Assume that the system is a monatomic gas ($c_\mathrm{V} = 1.5R$, i.e. $a = 0.4$) and that it is expanding in one step only from $P_\mathrm{i} = 4.0 \times 10^5$ Pa to $P_\mathrm{f} = 1.0 \times 10^5$ Pa. The entropy change, given by equation (13), is $\Delta S = 0.495nR$. But, for instance, if ten reservoirs are involved, equation (20), with $N = 10$, gives $\Delta S = 0.057nR$. It is not difficult to convince oneself that the addition of more and more work reservoirs makes $\Delta S \approx 0$.

If instead we had considered a compression from $P_\mathrm{i} = 1.0 \times 10^5$ Pa to $P_\mathrm{f} = 4.0 \times 10^5$ Pa, we would obtain $\Delta S[N = 1] = 0.585nR$ and $\Delta S[N = 10] = 0.058nR$.

4. Expanding (or compressing) in an arithmetic progression

Here we reconsider the problem of the previous section, but now the reservoir pressures are in an arithmetic progression. In this case, the pressure of each reservoir varies by a fixed quantity with respect to the previous one. This fixed quantity is $\varepsilon = (P_\mathrm{f} - P_\mathrm{i})/N$. Then, the pressure of reservoir k is given by $P_k = P_\mathrm{i} + k\varepsilon$. As before, an expansion corresponds to having $P_\mathrm{f} < P_\mathrm{i}$ and a compression to having $P_\mathrm{f} > P_\mathrm{i}$.

When the system at pressure P_{k-1} is put into contact with the reservoir at pressure P_k, the entropy change in the system is again given by (13) which leads to

$$\frac{\Delta S_k}{nR} = \frac{1}{a}\ln\left[1 + a\left(\frac{P_k}{P_{k-1}} - 1\right)\right] - \ln\left(\frac{P_k}{P_{k-1}}\right). \tag{24}$$

Therefore, considering N work reservoirs the entropy change is

$$\frac{\Delta S}{nR} = \frac{1}{a}\sum_{k=1}^{N}\ln\left(1 + a\frac{P_\mathrm{f} - P_\mathrm{i}}{NP_{k-1}}\right) - \sum_{k=1}^{N}\ln\left(\frac{P_k}{P_{k-1}}\right). \tag{25}$$

The second sum on the right-hand side of the above equation is quite easy to evaluate:

$$\sum_{k=1}^{N}\ln\left(\frac{P_k}{P_{k-1}}\right) = \ln\frac{P_1 P_2 P_3 \cdots P_{N-1} P_\mathrm{f}}{P_\mathrm{i} P_1 P_2 P_3 \cdots P_{N-1}} = \ln\left(\frac{P_\mathrm{f}}{P_\mathrm{i}}\right). \tag{26}$$

Thus, equation (25) can be simply written as

$$\frac{\Delta S}{nR} = \frac{1}{a}\sum_{k=1}^{N}\ln\left(1 + a\frac{P_\mathrm{f} - P_\mathrm{i}}{NP_{k-1}}\right) - \ln\left(\frac{P_\mathrm{f}}{P_\mathrm{i}}\right). \tag{27}$$

If the number of work reservoirs becomes arbitrarily large, the entropy change is obtained by considering the limit of the above sum as $N \to \infty$, i.e.

$$\begin{aligned}\lim_{N\to\infty}\left(\frac{\Delta S}{nR}\right) &= \frac{1}{a}\lim_{N\to\infty}\sum_{k=1}^{N}\ln\left(1 + a\frac{P_\mathrm{f} - P_\mathrm{i}}{NP_{k-1}}\right) - \ln\left(\frac{P_\mathrm{f}}{P_\mathrm{i}}\right)\\ &= \frac{1}{a}\lim_{N\to\infty}\sum_{k=1}^{N}\Delta P_k\frac{N}{P_\mathrm{f} - P_\mathrm{i}}\ln\left(1 + a\frac{P_\mathrm{f} - P_\mathrm{i}}{NP_{k-1}}\right) - \ln\left(\frac{P_\mathrm{f}}{P_\mathrm{i}}\right).\end{aligned} \tag{28}$$

As N becomes arbitrarily large, the above sum becomes an integral and, since

$$\lim_{N\to\infty} N\ln\left(1 + a\frac{P_\mathrm{f} - P_\mathrm{i}}{NP_{k-1}}\right) = a\frac{P_\mathrm{f} - P_\mathrm{i}}{P}, \tag{29}$$

we obtain

$$\lim_{N\to\infty}\left(\frac{\Delta S}{nR}\right) = \frac{1}{a}\int_{P_\mathrm{i}}^{P_\mathrm{f}}\frac{\mathrm{d}P}{P_\mathrm{f} - P_\mathrm{i}}\frac{a(P_\mathrm{f} - P_\mathrm{i})}{P} - \ln\left(\frac{P_\mathrm{f}}{P_\mathrm{i}}\right) = 0. \tag{30}$$

Similar to the case of the previous section, the above result shows that when infinitely many work reservoirs are involved, the system entropy change is again exactly zero.

Following the numerical example given for the geometric progression, from equation (27) with $N = 10$, we now have for the expansion $\Delta S\,[N = 10] = 0.066\,nR$ and for the compression $\Delta S\,[N = 10] = 0.068nR$. Comparing this result with that obtained if the reservoirs are in a geometric progression, we see that the relevant parameter is the number of intermediate reservoirs used to bring the system from P_i to P_f and not the way in which the intermediate pressures change. This is more clearly seen in the graph of figure 4, in the following section.

5. The impossibility of an adiabatic irreversible cycle

In general, a system can undergo reversible or irreversible cyclic processes. However, as already mentioned, it is impossible to build an irreversible cycle using only adiabatic processes. For instance, if we expand adiabatically and abruptly a system from P_i to P_f followed by an

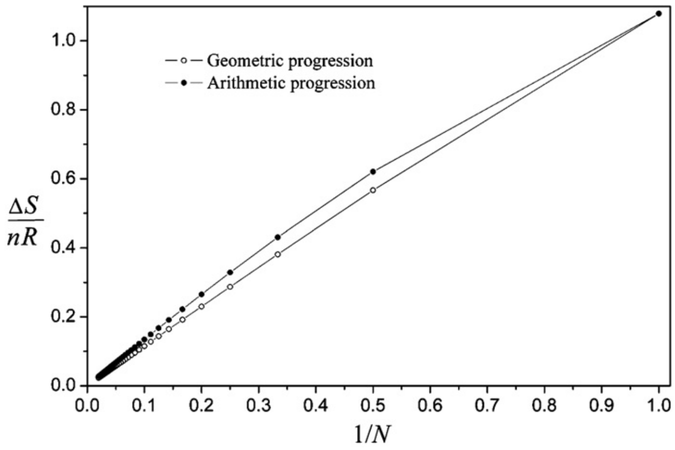

Figure 4. The change in entropy for a monatomic gas is shown in terms of the number of work reservoirs, N, for a process consisting of an expansion ($P_\mathrm{i} \rightarrow P_\mathrm{f}$) followed by a compression leading the system back to its initial pressure ($P_\mathrm{f} \rightarrow P_\mathrm{i}$). The reservoir pressures are in a geometric progression (equation (20)) or in an arithmetic progression (equation (27)). In both cases, the initial pressure is $P_\mathrm{i} = 4.0 \times 10^5$ Pa, then the gas is expanded to $P_\mathrm{f} = 1.0 \times 10^5$ Pa and finally it is again compressed to $P_\mathrm{i} = 4.0 \times 10^5$ Pa.

adiabatic compression from P_f to P_i, by using a finite number of steps in both cases, we cannot get the initial state because, according to (13), the entropy increases in all steps, as is clear from the graph of figure 2. In other words, to build a cycle resorting exclusively to adiabatic processes, all these processes should be reversible.

Therefore, we cannot study adiabatic cyclic processes involving a finite number of work reservoirs. However, we can consider expansions followed by compressions leading the system back to its initial pressure, via N reservoirs in geometric or arithmetic progressions. This is shown in the graph of figure 4, for a monatomic ideal gas.

It is clear that the entropy change diminishes rapidly as the number of work reservoirs increases. Obviously, for $N = 1$ a sequence of reservoirs is meaningless. For $N > 1$, when the number of reservoirs is small, the variation in entropy depends appreciably on the way in which the intermediate pressures change. For instance, for the case of two reservoirs ($N = 2$), the value of $\Delta S/(nR)$ shown in figure 4 increases by 0.054 from the geometric to the arithmetic progression. However, as the number of work reservoirs increases considerably, the way in which the intermediate pressures change becomes irrelevant in relation to the number of reservoirs. In the limit of an infinite number of reservoirs, the process becomes cyclic as well as reversible.

6. Conclusions

In this study, a novel expression which describes the change in entropy resulting from an abrupt adiabatic process in an ideal gas was used to deal with irreversible adiabatic expansions or compressions in order to get a new insight into the concepts of reversibility and entropy. It was shown that for exchanges of work involving an arbitrarily large number of work reservoirs, the system entropy change is exactly zero, i.e. the process becomes reversible. This was explicitly evaluated in two different situations: work reservoirs in a geometric progression and in an arithmetic progression. It was found that as the number of reservoirs increases more and more, the kind of progression becomes unimportant in relation to the number of reservoirs.

In going beyond the usual textbook explanations, it is hoped that this study will provide further insights and help to clarify the concepts of the reversible process and entropy, and therefore be relevant from a pedagogical and educational standpoint.

References

[1] Anacleto J and Anacleto J A C 2008 Thermodynamical interactions: subtleties of heat and work concepts *Eur. J. Phys.* **29** 555–66

[2] Anacleto J, Ferreira J M and Anacleto A 2008 Identical thermodynamical processes and the generalization of the Clausius inequality *Can. J. Phys.* **86** 369–77

[3] Anacleto J and Ferreira J M 2008 Heat and work concepts: are they invariant under a system–surroundings interchange? *Quim. Nova* **31** 1881–4

[4] Anacleto J and Ferreira J M 2008 Surroundings-based and system-based heat and work definitions: which one is the most suitable? *J. Chem. Thermodyn.* **40** 134–5

[5] Mungan C E 2007 Thermodynamics of a block sliding across a frictional surface *Phys. Teach.* **45** 288–91

[6] Gislason E A and Craig N C 2005 Cementing the foundations of thermodynamics: comparison of system-based and surroundings-based definitions of work and heat *J. Chem. Thermodyn.* **37** 954–66

[7] Mungan C E 2003 Irreversible adiabatic compression of an ideal gas *Phys. Teach.* **41** 450–53

[8] Besson U 2003 The distinction between heat and work: an approach based on a classical mechanical model *Eur. J. Phys.* **24** 245–52

[9] Craig N C and Gislason E A 2002 First law of thermodynamics; irreversible and reversible processes *J. Chem. Educ.* **79** 193–9

[10] Miranda E N 2000 When an irreversible cooling (or heating) becomes reversible *Eur. J. Phys.* **21** 239–43

[11] Styer D F 2000 Insight into entropy *Am. J. Phys.* **68** 1090–6

[12] Callen H B 1985 *Thermodynamics and an Introduction to Thermostatistics* (New York: Wiley)

[13] Anacleto J and Pereira M G 2009 From free expansion to abrupt compression of an ideal gas *Eur. J. Phys.* **30** 177–83

[14] Miranda E N 2008 What lies between a free adiabatic expansion and a quasi-static one? *Eur. J. Phys.* **29** 937–43

[15] Güémez J, Fiolhais C and Fiolhais M 2007 Physics of the fire piston and the fog bottle *Eur. J. Phys.* **28** 1199–205

[16] Zemansky M W and Dittman R H 1997 *Heat and Thermodynamics* 7th edn (Auckland: McGraw-Hill)

[17] Sears F W and Salinger G L 1982 *Thermodynamics, Kinetic Theory, and Statistical Thermodynamics* 3rd edn (New York: Addison-Wesley)